School Mathematics

Volume 1

Elementary Mathematics for High Schools

Includes brief review notes, worked examples, and test questions with answers.

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Preface

The volumes in the School Mathematics series contain brief review notes, selected questions with detailed solutions, and test questions with answers. We hope you find the material here useful.

Some questions in Volume 1 have been taken from the book *Integrated Mathematics for Explorers by Adeline Ng and Rajesh R. Parwani*, and the solutions from the accompanying *Solutions Manual by Chee Leong Ching and Sun Jie.*

Singapore, May 2015.

Chapter 1

Brief Review Notes

Consider the repeated multiplication $5 \times 5 \times 5$. A shorthand for that product is 5^3, a notation that is not only compact but also allows for rapid calculations using the rules that we summarise below.

More generally, if one writes $y = b^x$ for $b \neq 1$ and $b > 0$, then b is called the **base** while x is the **exponent** which need not be an integer. The function $y = b^x$ is sometimes referred to as an indicial function and the exponent x as the index or power.

Integral exponents occur in our familiar decimal notation which uses the base 10. For example, 352 is $3 \times 10^2 + 5 \times 10^1 + 2 \times 10^0$. That same number may be written in **scientific notation (standard form)** as 3.52×10^2.

More generally, any non-zero number may be written in the form $\pm A \times 10^p$ where $1 \leq A < 10$ and p is an integer.

Fractional exponents are useful too. For example $2^{1/2} = \sqrt{2}$.

Given a quadratic equation $ax^2 + bx + c = 0$, its roots may be found by completing the square: Multiply the equation by $4a$, add b^2 to both sides and rearrange to get $4a^2x^2 + 4abx + b^2 = b^2 - 4ac$, or $(2ax + b)^2 = \Delta$ where

$$\Delta \equiv b^2 - 4ac \tag{1.1}$$

is called the **discriminant**. The solutions are therefore given by

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}. \tag{1.2}$$

The two roots are real if and only if $\Delta \geq 0$; the case $\Delta = 0$ corresponds to a repeated root.

A different way of completing the square is often useful: $ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$. This form shows that the quadratic equation has its extremal value at $x = -b/(2a)$, the point being a maximum if $a < 0$ and a minimum if $a > 0$ (this is discussed more below).

What does the curve $y = ax^2 + bx + c$ look like? It is determined by the sign of a: When $a > 0$, we see that y is positive and increasing for very large values of $|x|$, so the curve must have a minimum point. Next, by completing the square, or equivalently by looking at the discriminant, we can determine if the curve has any intersection with the $y = 0$ axis (that is, any real roots). Similarly, the case $a < 0$ leads to a “hump” shaped curve which reaches a maximum value.

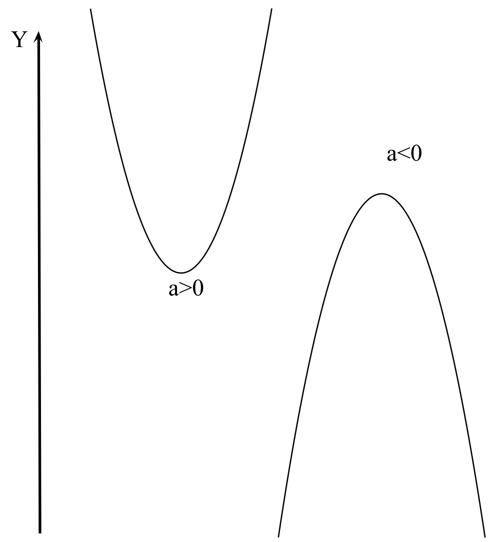

Figure 1.1: The shape of the parabola $y = ax^2 + bx + c$.

1.1 Relations and Properties

We list here some useful identities involving exponents.

For $a, b > 0$,

$$a^{-1} = \frac{1}{a}\,. \tag{1.3}$$

$$a^0 = 1\,. \tag{1.4}$$

$$a^{xy} = (a^x)^y\,. \tag{1.5}$$

$$a^x a^y = a^{x+y}\,. \tag{1.6}$$

$$(ab)^x = a^x b^x\,. \tag{1.7}$$

It is useful to note that for $a > 1$, $y = a^x$ is positive and an increasing function of x along the real line.

Chapter 2

Worked Examples

1. Express the following numbers in scientific notation:

   (a) -0.0031.

   (b) 123.456.

   (c) $3.23 \times 12.56 \times 0.02$.

   (d) $-24.31 \times 10^5 \times 0.07 \times 10^{-6}$.

– –

Solutions:
Recall that any non-zero number may be written in scientific notation: $\pm A \times 10^p$ where $1 \leq A < 10$ and p is an integer.

We have,

(a) $-0.0031 = -3.1 \times 10^{-3}$.
(b) $123.456 = 1.23456 \times 10^2$.
(c) $3.23 \times 12.56 \times 0.02 = 8.11376 \times 10^{-1}$.
(d) $-24.31 \times 10^5 \times 0.07 \times 10^{-6} = -1.7017 \times 10^{-1}$.

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2. The distances of the Moon, Sun and the star Proxima Centauri from Earth are respectively 3.8×10^5, 1.5×10^8 and 4.0×10^{13} kilometres (km). Taking the speed of light to be 3.0×10^5 km/s, calculate the time it takes light to reach Earth from each of those bodies. Express your answer in seconds for the Moon, in minutes for the Sun and in years for Proxima Centauri.

– –

Solutions:
Recall that speed (v) is defined as the ratio of distance travelled to the time taken for the journey, $v = d/t$. Therefore $t = d/v$.

For the Moon-Earth case, the time taken is

$$\begin{aligned} t_{\text{moon}} &= \frac{3.8 \times 10^5 \text{ km}}{3.0 \times 10^5 \text{ km/s}} \\ &= \frac{3.8}{3} \text{ s} \\ &= 1.3 \text{ s.} \end{aligned}$$

By the same method,

$$\begin{aligned} t_{\text{sun}} &= \frac{1.5 \times 10^8 \text{ km}}{3.0 \times 10^5 \text{ km/s}} \\ &= \frac{1.5}{3.0} \times 10^{8-5} \text{ s} \\ &= 0.5 \times 10^3 \text{ s} \\ &= 500 \text{ s.} \end{aligned}$$

Since 1 min = 60 s, therefore

$$\begin{aligned} t_{\text{sun}} &= \frac{500}{60} \text{ min} \\ &= 8.3 \text{ min.} \end{aligned}$$

Similarly,

$$\begin{aligned} t_{\text{centauri}} &= \frac{4.0 \times 10^{13} \text{ km}}{3.0 \times 10^5 \text{ km/s}} \\ &= \frac{4}{3} \times 10^{13-8} \text{ s} \\ &= \frac{4}{3} \times 10^8 \text{ s.} \end{aligned}$$

Now, $60 \times 60 = 3600$ seconds make one hour, 24 hours make a day and 365 days make a year. Therefore

$$\begin{aligned} t_{\text{centauri}} &= \frac{\frac{4}{3} \times 10^8 \text{ s}}{365 \times 24 \times 60 \times 60 \text{s/yr}} \\ &= 4.2 \text{ yrs.} \end{aligned}$$

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3. Re-arrange each of the following sequences in order, from the smallest to the largest, without using a calculator. Justify your answers.

(a) $7^5,\ 5^{7/2},\ 7^7,\ 5^5$.

(b) $(2^6)^{\frac{1}{3}},\ \left(2^{\frac{1}{2}}\right)^{-4},\ 2^2 \times 2^{-3},\ (2^2 \times 3^4)^{-\frac{1}{2}}$.

– –

Solutions:
Recall that a^x is an increasing function of x for $a > 1$.

(a) We have $7^7 > 7^5$, and similarly $5^5 > 5^{7/2}$. Furthermore, $5^5 < 7^5$ since $1 < \left(\frac{7}{5}\right)^5$. Therefore we can arrange the terms in increasing order as

$$5^{7/2} < 5^5 < 7^5 < 7^7.$$

(b) *Recall that $(a^x)^y = a^{xy}$ and $a^x \times a^y = a^{x+y}$.*
We first rewrite all the numbers in same base,

$$\begin{aligned}
(2^6)^{1/3} &= 2^2\ . \\
(2^{1/2})^{-4} &= 2^{-2}\ . \\
2^2 \times 2^{-3} &= 2^{2-3} = 2^{-1}\ . \\
(2^2 \times 3^4)^{-1/2} &= (2^2)^{-1/2} \times (3^4)^{-1/2} \\
&= 2^{-1} \times 3^{-2}.
\end{aligned}$$

We have $2^{-2} < 2^{-1} < 2^2$. Also, $2^{-1} \times 3^{-2} < 2^{-1} \times 2^{-1} = 2^{-2}$. Therefore,

$$(2^2 \times 3^4)^{-1/2} < (2^{1/2})^{-4} < 2^2 \times 2^{-3} < (2^6)^{1/3}.$$

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4. Simplify the following expression as much as possible without using a calculator: $\left(2x^2\right)^{-\frac{1}{2}}\ (3x^{-5})^{-1}$.

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Solution:
Recall $(ab^x)^y = a^y b^{xy}$.
Therefore,

$$\begin{aligned}
(2x^2)^{-1/2}\ (3x^{-5})^{-1} &= 2^{-1/2} \times x^{-1} \times 3^{-1} \times x^5 \\
&= \frac{1}{3\sqrt{2}}\ x^4.
\end{aligned}$$

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5. Solve each of the following equations for x without using a calculator:

   (a) $5^{2x-1} = 1/25$.

   (b) $3^{2-x}(2^2 \times 3^{2x+1}) = 4/9$.

Solutions:
Recall $a^x = a^y \Rightarrow x = y$.

(a) We write all terms in the same base and compare exponents:

$$\begin{aligned} 5^{2x-1} &= \frac{1}{25} \\ &= \frac{1}{5^2} \\ &= 5^{-2} \\ \Rightarrow 2x - 1 &= -2 \\ \therefore x &= -\frac{1}{2}. \end{aligned}$$

(b) Similarly,

$$\begin{aligned} 3^{2-x}(2^2 \times 3^{2x+1}) &= \frac{4}{9} \\ &= 2^2 \times 3^{-2} \\ 3^{2-x} \times 3^{2x+1} &= 3^{-2} \\ 3^{(2-x)+(2x+1)} &= 3^{-2} \\ 3^{x+3} &= 3^{-2} \\ \Rightarrow x + 3 &= -2 \\ \therefore x &= -5. \end{aligned}$$

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6. A bank pays 3% compound interest per month on its fixed deposits. If \$ 10,000 is deposited, what would be the accumulated amount three months later?

Solution:
Note: Compounding at that rate means that at the end of each month the total amount would be 1.03 *times the amount at the start of that month. The total amount (principal plus accumulated interest), in dollars, at the end of* n *months is given by* $P(n) = (1 + 0.03)^n P_0$ *where* P_0 *is the initial amount deposited.*

For $n = 3$ we have

$$\begin{aligned} P &= (1+0.03)^3(10^4) \\ &= \left[1+3(0.03)+3(0.03)^2+(0.03)^3\right](10^4) \\ &= 10,927.27. \end{aligned}$$

Answer: \$ $10,927.27$.

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7. A certain bank pays $r\%$ simple interest for deposits which are kept for at least 3 months. If \$ $10,000$ was deposited with this bank for three months, what value of r would give the same return as the bank of the previous question?

− −

Solution:
The total accumulated amount would now be $P_2(n) = (1 + r/100)P_0$ where $P_0 = 10^4$ dollars is the initial amount deposited. We need

$$\begin{aligned} \left(1+\frac{r}{100}\right)(10^4) &= 10,927.27 \\ \left(1+\frac{r}{100}\right) &= 1.092727 \\ \frac{r}{100} &= 0.092727 \\ r &\approx 9.27\ . \end{aligned}$$

That is, the simple interest rate would have to be about 9.27%.

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8. Simplify the following expression:

$$\frac{(p^2+2)(p-3)+5p^2+10}{p^2+2p}$$

− −

Solution:
Let us simplify the denominator first, writing it as $p(p + 2)$. This suggests we try to find similar factors in the numerator. We notice $5p^2 + 10 = 5(p^2 + 2)$ which reveals the $p^2 + 2$ common to the first term of the numerator. This looks promising:

$$\begin{aligned} \frac{(p^2+2)(p-3)+5p^2+10}{p^2+2p} &= \frac{(p^2+2)(p-3)+5(p^2+2)}{p(p+2)} \\ &= \frac{(p^2+2)(p-3+5)}{p(p+2)} \\ &= \frac{(p^2+2)(p+2)}{p(p+2)} \\ &= \frac{(p^2+2)}{p}. \end{aligned}$$

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9. A symmetrical spherical shell is created by removing a central sphere of radius R from the inside of a solid metal sphere of radius $2R$.

   (a) What is the volume of the remaining spherical shell?

   (b) The shell from part (a) is melted and the metal used to construct a cone of circular base with radius R. Determine the height of the cone in terms of R.

– –

Solution:
Recall the volume of a sphere: $4\pi r^3/3$ *where* r *is the radius.*

(a) The volume of the shell is

$$\begin{aligned}\frac{4\pi}{3}\left((2R)^3 - R^3\right) &= \frac{4\pi}{3}\left(8R^3 - R^3\right) \\ &= \frac{28\pi}{3}R^3 \ .\end{aligned}$$

(b) The volume of the cone is (area of base) $\times$ (height)/3. Letting the height be h, we have

$$\begin{aligned}\frac{h}{3}\pi R^2 &= \frac{28\pi}{3}R^3 \\ \Rightarrow h &= 28R \ .\end{aligned}$$

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10. A line segment is divided into two parts of length a and b such that $\dfrac{a}{b} = \dfrac{a+b}{a} \equiv \phi$. Determine the numerical value of ϕ, known as the "Golden Ratio".

– –

Solution:
Note: $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Let $\dfrac{a}{b} = x$, we can then rewrite the equation in the question as

$$\begin{aligned}\frac{a}{b} &= \frac{a+b}{a} \\ &= 1 + \frac{a}{b} \\ \Rightarrow x &= 1 + \frac{1}{x}.\end{aligned}$$

Next, we solve for x,

$$\begin{aligned}
x - \left(1 + \frac{1}{x}\right) &= 0 \\
\frac{x^2 - x - 1}{x} &= 0 \\
\Rightarrow x^2 - x - 1 &= 0 \\
\therefore x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-1)}}{2} \\
&= \frac{1 \pm \sqrt{5}}{2}.
\end{aligned}$$

Since the "Golden Ratio" is defined as the ratio of lengths (positive quantities), it has to be positive definite. Hence we only accept the positive solution, $\phi = x = \dfrac{1 + \sqrt{5}}{2}$.

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11. Neo, is on a platform that is $y_0 = 10$ m above ground. He throws a ball vertically upwards from that point. The subsequent displacement of the ball relative to the ground is given by $y(t) = y_0 + ut - \frac{gt^2}{2}$ where u m/s is the initial vertical velocity of the ball, g m/s^2 the acceleration due to gravity and t the time in seconds. If $u = 5$ and $g = 10$,

    (a) Factorise the expression for $y(t)$ for the given constants and use it to find the time taken for the ball to reach the ground.

    (b) By writing $y(t)$ in the form $y = a(t + b)^2 + c$, determine the maximum height reached by the ball and the time taken for it to reach that height.

    (c) For how long did the ball stay 5 m or more above ground?

\- -

Solutions:

(a) With the given information, we have

$$y(t) = 10 + 5t - 5t^2,$$

which can be factorised into the form $y(t) = (t + 1)(-5t + 10)$.
When the ball reaches the ground again, $y(t) = 0$, so

$$\begin{aligned}
0 &= (t + 1)(-5t + 10) \\
\therefore t &= -1 \text{ s or } 2 \text{ s}.
\end{aligned}$$

Since t represents the time, which is positive, the only acceptable answer is $t = 2$ s.

(b) Let us complete the square,

$$\begin{aligned} y(t) &= -5t^2 + 5t + 10 \\ &= -5\big(t^2 - t - 2\big) \\ &= -5\left[(t-1/2)^2 - 1/4 - 2\right] \\ &= -5\left[(t-1/2)^2 - 9/4\right] \\ &= -5(t-1/2)^2 + 45/4. \end{aligned} \tag{2.1}$$

We see that the first term on the right hand side of (2.1) is never positive, $-5(t-1/2)^2 \le 0$. So, $y(t)$ will be maximum when that term vanishes. The maximum height is $y_{\text{max}} = 45/4$ m and it is reached at $t = 1/2$ s.

(c) The ball starts at $t = 0$ from a platform which is 10 m above ground. It will move upwards but will eventually fall towards the ground. To find when it reaches $y = 5$ m, we solve

$$\begin{aligned} 5 &= -5t^2 + 5t + 10 \\ 0 &= -5t^2 + 5t + 10 - 5 \\ 0 &= -5t^2 + 5t + 5 \\ 0 &= -5(t^2 - t - 1) \\ \Rightarrow 0 &= t^2 - t - 1 \\ \therefore t &= \frac{-(-1) \pm \sqrt{1 - 4(-1)}}{2} \\ &= \frac{1 \pm \sqrt{5}}{2}. \end{aligned}$$

Since $t \ge 0$, we have to choose $t = (1 + \sqrt{5})/2$ s.

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12. Solve the following equation for x: $\dfrac{1}{1-x} + \dfrac{1}{x+2} = 3.$

– –

Solution:
We combine all terms over one common denominator,

$$\begin{aligned} 3 &= \frac{1}{1-x} + \frac{1}{x+2} \\ 0 &= \frac{1}{1-x} + \frac{1}{x+2} - 3 \\ 0 &= \frac{(x+2) + (1-x) - 3(1-x)(x+2)}{(1-x)(x+2)} \\ 0 &= \frac{(x+2) + (1-x) - 3(-x^2 - x + 2)}{(1-x)(x+2)} \\ 0 &= \frac{3x^2 + 3x - 3}{(1-x)(x+2)}\ . \end{aligned}$$

Since the numerator must be zero,

$$\begin{aligned} \Rightarrow x^2 + x - 1 &= 0 \\ \therefore\ x &= \frac{-1 \pm \sqrt{1 - 4(-1)}}{2} \\ &= (-1 \pm \sqrt{5})/2\ . \end{aligned}$$

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13. Find the range of values of x for which:

 (a) $2 \geq 5x + x^2$.

 (b) $4x \geq (x-1)(x-3)$.

– –

Solutions:

(a) We begin by bringing all the terms to one side and factorising the quadratic expression using its roots.

$$\begin{aligned} 2 &\geq 5x + x^2 \\ 0 &\geq x^2 + 5x - 2. \end{aligned}$$

Let $f_1(x) = x^2 + 5x - 2$. Its roots are

$$x = \frac{-5 \pm \sqrt{25 - 4(-2)}}{2} = \frac{-5 \pm \sqrt{33}}{2}.$$

Let $x_2 = (-5 + \sqrt{33})/2$ and $x_1 = (-5 - \sqrt{33})/2$; see Fig. 2.1. Then

$$\begin{aligned} & 0 \geq (x - x_1)(x - x_2) \\ & \Rightarrow x_1 \leq x \leq x_2 \\ \therefore \quad & \frac{-5 - \sqrt{33}}{2} \leq x \leq \frac{\sqrt{33} - 5}{2}. \end{aligned}$$

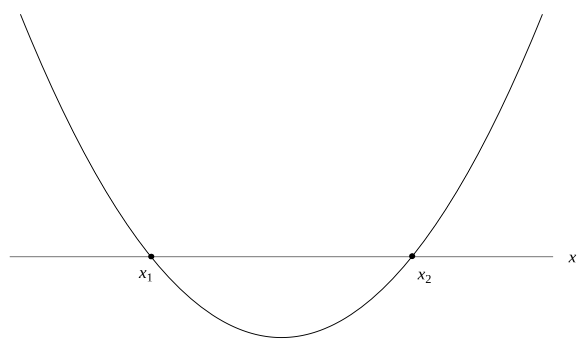

Figure 2.1: Parabola intersecting the x-axis.

(b) We proceed as in the previous example.

$$\begin{aligned} 4x &\geq (x-1)(x-3) \\ 0 &\geq (x-1)(x-3) - 4x \\ &\geq x^2 - 8x + 3 \end{aligned} \tag{2.2}$$

Let $f_2(x) = x^2 - 8x + 3$. The roots of $f_2(x)$ are

$$\begin{aligned} x &= \frac{8 \pm \sqrt{64 - 4(3)}}{2} \\ &= 4 \pm \sqrt{13}. \end{aligned}$$

Let $x_2 = 4 + \sqrt{13}$ and $x_1 = 4 - \sqrt{13}$. Thus,

$$\begin{aligned} & 0 \geq (x - x_1)(x - x_2) \\ & \Rightarrow x_1 \leq x \leq x_2 \\ \therefore \quad & 4 - \sqrt{13} \leq x \leq 4 + \sqrt{13}. \end{aligned}$$

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14. If $f(x) = x^2 + bx + c$, determine the constants b, c for the separate exercises below and sketch the curve $y = f(x)$:

(a) $f(x)$ has a double root at $x = 5$.

(b) $f(x)$ has roots at $x = -1$ and $x = 2$.

– –

Solutions:

(a) If $f(x)$ has a double root at $x = 5$, it implies that $f(x) = a(x-5)^2 = a(x^2 - 10x + 25)$, with a a constant. Comparing with the given expression, $f(x) = x^2 + bx + c$, we deduce that $b = -10,\ c = 25$.

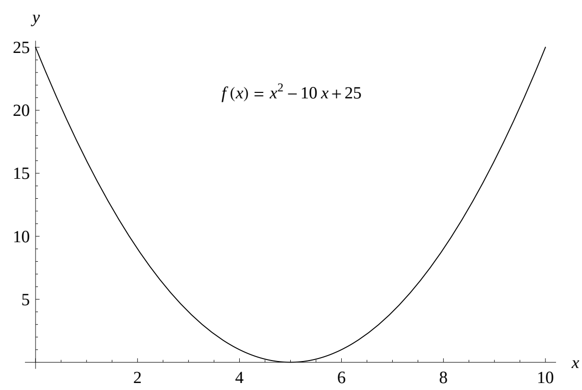

Figure 2.2: Plot of $f(x) = x^2 - 10x + 25$. There is a double root at $x = 5$.

(b) If $f(x)$ has roots at $x = -1$ and $x = 2$, it must be of the form $f(x) = a(x+1)(x-2) = a(x^2 - x - 2)$, with a a constant. Comparing with the given expression gives $b = -1,\ c = -2$.

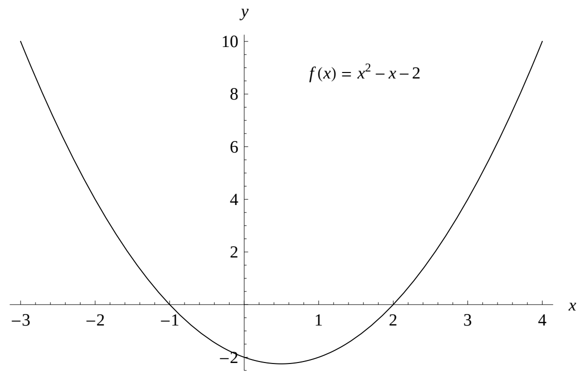

Figure 2.3: Plot of $f(x) = x^2 - x - 2$. The roots are at $x = -1$ and $x = 2$.

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15. For each $f(x)$ determined in the previous question, sketch $y = |f(x)|$ and determine the range that y takes for x between 0 and the positive root.

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Solutions:

(a) Since $f(x) = x^2 - 10x + 25 = (x-5)^2$ is always positive definite, the modulus graph is exactly same as the original graph, $|f(x)| = f(x)$, (see Fig.2.2). From the plot, the range that y takes for x between 0 and the positive root $x = 5$ is $0 \leq y \leq 25$.

(b) The plot of $|f(x)|$, where $f(x) = x^2 - x - 2$, is shown below.

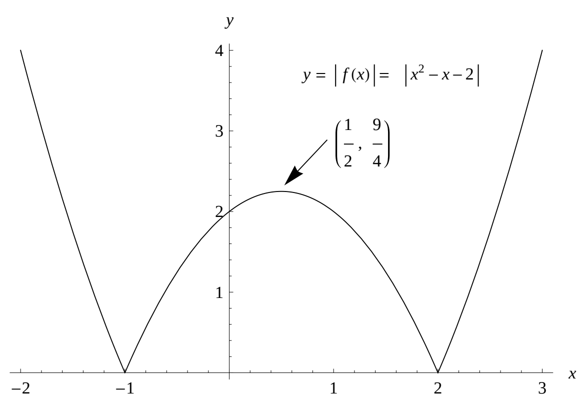

Figure 2.4: Plot of $|f(x)|$ where $f(x) = x^2 - x - 2$.

The local maximum of the graph $|f(x)|$ can be determined as follows,

$$\begin{aligned} f(x) &= |x^2 - x - 2| = |(x-1/2)^2 - 1/4 - 2| \\ &= |(x-1/2)^2 - 9/4|. \end{aligned}$$

Thus the maximum occurs at $x = 1/2$ and $y_{\text{max}} = 9/4$. The range that y takes for x between 0 and the positive root $x = 2$ is $0 \le y \le 9/4$.

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16. If $f(x) = x^2 + bx + 2b$, determine constraints on the constant b for the separate exercises below:

(a) $f(x)$ has no real roots.

(b) $f(x)$ has a root at $x = -1$.

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Solutions:
Recall that a quadratic equation $Ax^2 + Bx + C = 0$ has discriminant $\Delta = B^2 - 4AC$. The discriminant of $f(x) = x^2 + bx + 2b$ is $b^2 - 8b = b(b-8)$.

(a) For $f(x)$ to have no real roots, the discriminant must be negative,

$$\begin{aligned} & 0 > b(b-8) \\ \therefore \quad & 0 < b < 8. \end{aligned}$$

(b) If $f(x)$ has a root at $x = -1$, then $f(-1) = 0$, implying

$$\begin{aligned} 0 &= 1^2 + b(-1) + 2b \\ \Rightarrow b + 1 &= 0 \\ \therefore b &= -1. \end{aligned}$$

More Practice Questions
on these and other topics are in the book
Integrated Mathematics for Explorers by Adeline Ng and Rajesh R. Parwani.
The book also includes puzzles, challenges, and suggestions for investigation.

A *Solutions Manual by Chee Leong Ching and Sun Jie* is also available.

View sample pages at www.simplicitysg.net/books/imaths.

Chapter 3

Test Yourself

1. A solid sphere occupies a volume of exactly 5000 cm^3. Determine its surface area to three significant figures and express your answer in standard form.
2. In Worked Example (11) of Chap. 2, if Neo were to throw the ball the same way on the Moon, where the g value is 1/6-th of that on Earth, what would the answers to the various parts of the question be?
3. Re-arrange the following three numbers in order, from the smallest to the largest, without using a calculator: $8^{1/3}$, $4^{-\frac{3}{2}}$, $\left(\frac{1}{2}\right)^{-2}$. Justify your answer.
4. A bank pays 4% compound interest per month on its fixed deposits. If a man deposits \$ X, and accumulates \$ 21,500 at the end of 6 months, what is X? Express your answer to the nearest dollar.
5. Rewrite the expression below to obtain an explicit form for x in terms of the constants a and b. Factorise your expression where possible.
$$\frac{1}{x} = \frac{1}{a^2} - \frac{1}{b^2}\ .$$
6. Find the values of x for which $2x + 5 \leq (3 - x)^2$.
7. If $f(x) = x^2 + bx + 2b$, determine constraints on the constant b if $f(x)$ has two distinct real roots.
8. Challenge: If the sum of the first n even natural numbers is A, and the sum of the first n odd natural numbers is B, show that $A - B = n$. (Note: A natural number is a positive integer).

Chapter 4

Answers to Test

1. 1.41×10^3 cm^2.
2. (a) $t = (3 + \sqrt{21})$ s.
   (b) $y_{\text{max}} = 35/2$ m when $t = 3$ s.
   (c) $t = (3 + \sqrt{15})$ s.
3. $4^{-3/2} < 8^{1/3} < \left(\frac{1}{2}\right)^{-2}$.
4. \$ 16,992.
5. $\dfrac{(ab)^2}{(b-a)(b+a)}$.
6. $x \geq 2(2+\sqrt{3})$ or $x \leq 2(2-\sqrt{3})$.
7. $b > 8$ or $b < 0$.
8. You can pair consecutive integers in the difference, for example, for $n = 3$, $(2+4+6)-(1+3+5) = (2-1)+(4-3)+(6-5) = 1+1+1 = 3$, and generalise. Or, use the formula for the sum of an arithmetic progression.

Did You Know?

The sum of the first n odd integers is n^2 .

For example, $1 + 3 + 5 = 3^2$.

Can you prove the general result below?

$$\sum_{k=1}^{n}(2k-1) = n^2 \ .$$