

*‘Everything should be made as simple as possible, but not simpler.’*

— attributed to Albert Einstein.

## **Simplicity in Complexity**

### An Introduction to Complex Systems

How do scientists model crowd behaviour, epidemics, earthquakes or the internet?

What can we learn from the collective intelligence and adaptability of an ant colony?

This book answers such questions by highlighting common themes in the study of complex systems.

Topics covered include self-organisation, emergence, agent-based simulations, complex networks, phase plane plots, fractals, chaos, measures of complexity, model building, and the scientific method. Explanations are simple and concise, with common misconceptions clarified. Numerous exercises help enthusiasts consolidate their understanding through peer learning.

Supplementary resources are at the companion websites [www.simplicitysg.net/books](http://www.simplicitysg.net/books) and [www.facebook.com/simcomty](http://www.facebook.com/simcomty).

About the Author:

Dr. Parwani tested the modules ‘Simplicity’ and ‘Complexity’ for more than a decade on willing undergraduates in a multi-disciplinary programme. This book summarises the content of those experiments.

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by Adeline Ng and Rajesh R. Parwani

*Real World Mathematics*  
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# Simplicity in Complexity

An Introduction to Complex Systems

Rajesh R. Parwani



Simplicity Research Institute, Singapore  
[www.simplicitysg.net](http://www.simplicitysg.net)

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## Simplicity in Complexity

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# Preface

A working hypothesis of scientists is that there exist relatively simple causes underlying all phenomena, no matter how complex.

Roughly speaking, one says that a system is complex if it consists of many interacting components. The complexity that is usually observed arises from the size of the system (the number of components) rather than the interaction rules.

The title of this book summarises the two ways that simplicity<sup>1</sup> can manifest itself: Simplicity of the underlying rules of a complex system and simplicity of emergent laws and structures.

We will see how relatively simple rules, principles and methodologies can be applied to study systems ranging from the domain of physics to other fields such as chemistry, biology, ecology and sociology.

In this book, we will not focus on a detailed study of individual complex systems, but on uncovering common themes that can be used in our understanding of different systems. This book may, therefore, be used for an introductory course on Complex Systems.

The exercises at the end of each chapter are meant to encourage active learning through reading, thinking, experimentation, and discussion with other emergent beings.

We welcome feedback and questions from users of this book; please email us at **enquiry@simplicitysg.net**. Updates will be posted on the book's webpage at

[www.simplicitysg.net/books](http://www.simplicitysg.net/books)

Some parts of the 'Model Building' and 'Dynamical Systems'

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<sup>1</sup>'Simplicity' does not necessarily mean 'easy to understand'. However, in this book we will attempt to keep things easy.

chapters had previously appeared in another book by one of the current authors: ‘Real World Mathematics’ by W.K. Ng and R. Parwani.

## 0.1 Conventions

Key concepts are highlighted in italics while footnotes provide clarification or commentary. Single ‘quotes’ focus on particular words.

Where possible, we have attempted to locate the original sources for the ideas summarised in each chapter, and placed those references at the end of the chapter together with accessible secondary references<sup>2</sup>.

## 0.2 Acknowledgements

I am grateful to the hundreds of students who participated in the ‘Simplicity’ and ‘Complexity’ modules between 2001 and 2012, helping to evolve the material that has been partly condensed into this book.

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Rajesh R. Parwani

Jan 2015, Singapore.

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<sup>2</sup>Readers who notice any omissions are welcome to provide us with feedback.

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specialises in quantum theory and cosmology. He has mentored research students from high-school to graduate school, and taught university level modules in quantitative reasoning, multi-disciplinary science and physics.

His other books are *Integrated Mathematics for Explorers* (with Adeline Ng) and *Real World Mathematics* (with Dr. W.K. Ng).

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